Solutions to JEE Advanced Home Practice Test -4 |JEE 2024 | Paper-2

PHYSICS

1.(ABCD) $N_2 - mg = 0 \quad \Rightarrow \quad N_2 = mg$

$N_1 = ma\,; \qquad N_1 + ma = m\left(\dfrac{v^2}{R}\right)$

$N_1 + ma = m2g \quad \Rightarrow \quad N_1 = mg$

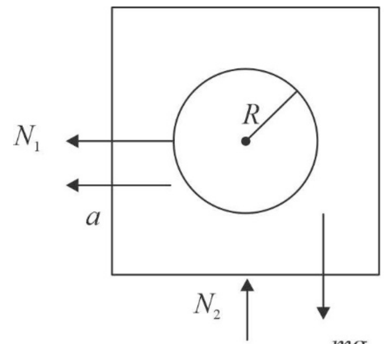
And $a = g$

$\vec{a}_{P.g} = \vec{a}_{P.C} + \vec{a}_{C.g}$

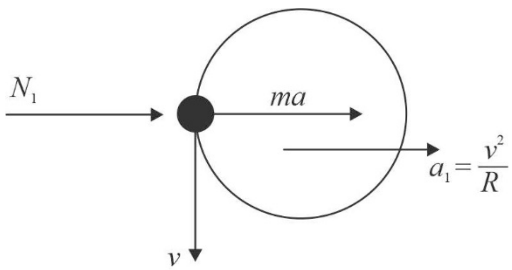
$\to g$

$\downarrow \; = g\sqrt{2}$

g



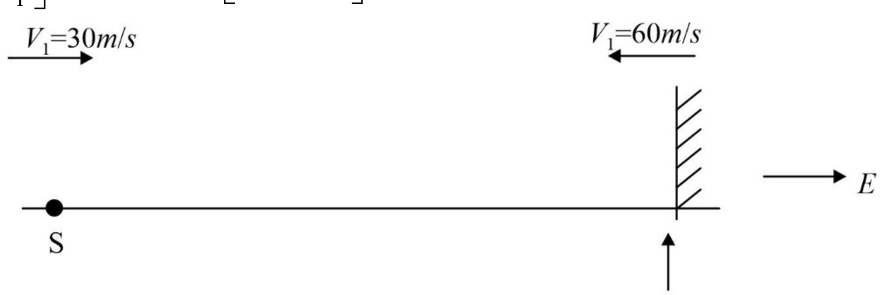
F.B.D. of cube



F.B.D of particle w.r.t. cube

2.(AD) Frequency received by the reflector is

$$f = f_0\left[\frac{V+V_2}{V-V_1}\right] \qquad = 1200\left[\frac{330+60}{330-60}\right] = 1200 \times \frac{390}{300} = 1560\ Hz$$



$$\lambda_1 = \frac{V-V_1}{f_0} = \frac{330-30}{1200} = \frac{1}{4}m$$

$$\lambda_2 = \frac{V-V_2}{f} = \frac{330-60}{1560} = \frac{27}{156} = \frac{9}{52}m \qquad \therefore \qquad \frac{\lambda_1}{\lambda_2} = \frac{1}{4} \times \frac{52}{9} = \frac{13}{9}$$

3.(B) $(1)\sin\theta = n(\lambda)\sin\theta'$

So, $$0 = \sin\theta'\frac{dn(\lambda)}{d\lambda} + n(\lambda)\cos\theta'\frac{d\theta'}{d\lambda}$$

$$\therefore \quad d\theta' = -\frac{\sin\theta'}{\cos\theta'}\frac{1}{n(\lambda)}\frac{dn(\lambda)}{d\lambda}d\lambda \qquad \therefore \quad \delta\theta' = -\frac{\tan\theta'}{n(\lambda)}\frac{dn(\lambda)}{d\lambda}\delta\lambda$$

4.(AD)

Consider FBD of sphere B as shown in Fig.1.

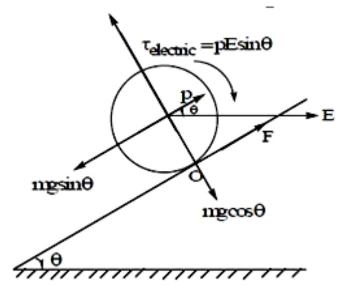


Fig.1.

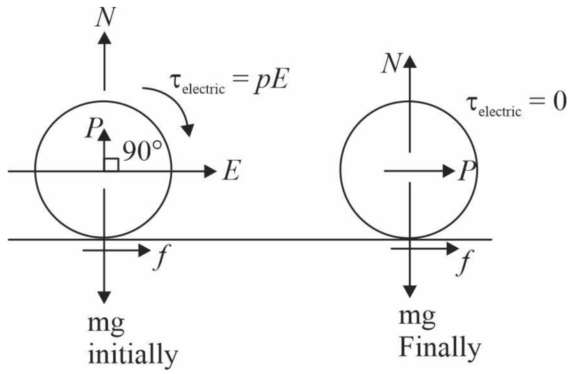
About contact point O of the sphere with the incline,

τ_1 = torque of $mg\sin\theta = mg\sin\theta.r$ (anti-clockwise)

τ_1 = torque of electric field E due to infinite sheet $= pE\sin\theta$, (anti-clockwise)

For equilibrium of B, $\tau_1 = \tau_2 \Rightarrow mg\sin\theta.r = pE\sin\theta$

$\Rightarrow mgr = pE$ … (i)



Moment of inertial of the sphere about O, using parallel axes theorem,

$$I = I_{CM} + md^2 = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2 \quad \text{....(ii)}$$

Torque of electric force of sheet on dipole, $\tau = pE\sin 90° = pE$ ....(iii)

Using Equation (iii) and (ii), We get

Angular acceleration of the sphere is $\alpha = \frac{\tau}{1} = \frac{pE}{\frac{7}{5}mr^2} = \frac{5pE}{7mr^2}$

By condition of rolling without slipping, $a = \alpha r = \frac{5pE}{7mr} = \frac{5mgr}{7mr}$

[using Equation (i)] $\Rightarrow a = \frac{5g}{7}$ ……(iv) $\therefore$ Option (a) is correct.

From FBD in Fig.2, using, $F = ma$ in horizontal direction, We get $f = m\left(\frac{5g}{7}\right) = \frac{5mg}{7}$

[using Eq. (iv)] $\therefore$ Option (b) is incorrect.

Due to electric field torque experienced by sphere A, will rotate the dipole clockwise. Potential energy of a dipole is given by $U = -pE\cos\theta$

Therefore, $U_{initial} = -pE\cos 90° = 0$ ……(v)

$U_{final} = -pE\cos 0° = -pE$ ……(vi)

Considering, rotation of sphere by $\frac{\pi}{2}$,

Gain in KE = Loss in $PE \frac{1}{2}I\omega^2 = U_{initial} - U_{final}$

$$\frac{1}{2}\cdot\frac{7}{5}mr^2\omega^2 = 0-(-pE) \Rightarrow \omega = \sqrt{\frac{10pE}{7mr^2}} = \sqrt{\frac{10p\left(\frac{\sigma}{2\in_0}\right)}{7mr^2}} = \sqrt{\frac{5\rho\sigma}{7mr^2\in_0}}$$

$\Rightarrow$ Option (C) is incorrect.

Finally, When dipole is parallel to horizontal surface, torque due to electric force is equal to

$\tau = pE\sin 0^0 = 0 \Rightarrow \alpha_{CM} = \frac{\tau}{1} = 0 \Rightarrow f = ma_{CM} = 0 \;\therefore$ Option (d) is correct.

5.(BCD)

(A) Heat supplied in isochoric process AB , $Q_{AB} = W_{AB} + \Delta U_{AB}, [W_{AB} = 0, as\, \Delta V = 0]$

$$Q_{AB} = 0 + \frac{f}{2}nR(T_B - T_A); \quad Q_{AB} = \frac{3R(T_B - T_A)}{2}$$

option (A) is incorrect

(B) Total heat supplied to gas.

$$Q = Q_{AB} + Q_{BC} = \frac{3R(T_B - T_A)}{2} + W_{BC} + \Delta U_{BC}$$

$$Q = \frac{3R(T_B - T_A)}{2} + \int PdV + \frac{f}{2}nR(T_C - T_B)$$

we have given, $P = K\sqrt{T} \quad \Rightarrow \frac{nRT}{V} = K\sqrt{T}$

$$V = \frac{nR\sqrt{T}}{K} \quad \Rightarrow dV = \frac{nRdT}{2K\sqrt{T}}$$

Now, $Q = \frac{3R(T_B - T_A)}{2} + \int_{T_B}^{T_C} K\sqrt{T} \times \frac{nRdT}{2K\sqrt{T}} + \frac{3}{2}R(T_C - T_B)$

$$Q = \frac{3R(T_B - T_A)}{2} + \frac{R}{2}[T_C - T_B] + \frac{3}{2}R(T_C - T_B)$$

$$Q = \frac{R}{2}[3T_B - 3T_A + T_C - T_B + 3T_C - 3T_B]; \quad Q = \frac{R(4T_C - T_B - 3T_A)}{2}$$

(C) $W = \int PdV = \frac{R}{2}(T_C - T_B)$

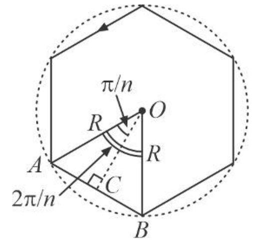
(D) For BC process, for process AB ,

$P\alpha\sqrt{T}$ $\qquad \frac{P_A}{T_A} = \frac{P_B}{T_B}$

$\frac{P_B}{P_C} = \sqrt{\frac{T_B}{T_C}}$ $\qquad P_B = \frac{T_B}{T_A}P_A$

$T_C = \frac{T_B P_C^2}{P_B^2}$ $\qquad T_C = \frac{T_B P_C^2}{\frac{T_B^2}{T_A^2}P_A^2} = \frac{T_A^2}{T_B}\frac{P_C^2}{P_A^2}$

6.(AC) The situation described in question is shown in figure.



In figure, OC is the perpendicular distance of one segment of polygon from the centre. Here $\angle AOB = (2\pi/n)$ as there are n elements like AB thus distance OC is given as

$$OC = R\cos(\pi/n)$$

The magnetic induction at O due to a straight current carrying element AB is given as

$$B_1 = \frac{\mu_0 i}{4\pi} \times \frac{1}{R\cos(\pi/n)}\left[\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{\pi}{n}\right)\right]$$

$$\Rightarrow \quad B_1 = \frac{\mu_0 i}{4\pi} \times \frac{1}{R\cos(\pi/n)} \times 2\sin\left(\frac{\pi}{n}\right) \quad \ldots \text{(i)}$$

As there are n sides in the polygon, the total magnetic induction due to polygon is given as

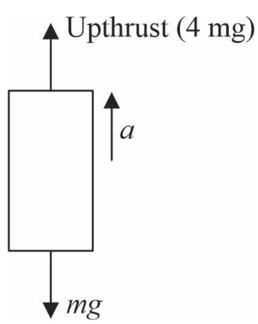
$$B = \frac{\mu_0 ni}{2\pi R}\tan\left(\frac{\pi}{n}\right) \quad \ldots \text{(ii)}$$

When $n \to \infty, \tan(\pi/n) \approx \pi/n$ which gives

$$B = \frac{\mu_0 ni}{2\pi R}\left(\frac{\pi}{n}\right) = \frac{\mu_0 i}{2R} \quad \ldots \text{(iii)}$$

Above expression in equation (iii) is a result of magnetic induction due to a circular coil as a polygon with infinite sides transforms into a circular coil.

7.(3)

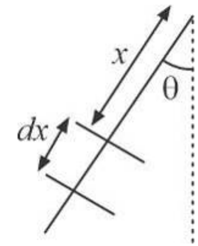


8.(1.50) The density of liquid is four times that of cylinder, hence in equilibrium position one fourth of the cylinder is submerged.

So as the cylinder is released from initial position, it moves by $\frac{3l}{4}$ to reach its equilibrium position. The upward motion in this time in SHM. Therefore required velocity is $v_{\max} = \omega A . \omega = \sqrt{\frac{4g}{l}}$ and $A = \frac{3l}{4}$.

Therefore $v_{\max} = \frac{3}{2}\sqrt{gl}$

9.(6) Consider an element of rod.
Consider an element of rod.



$$dL = x \times x \times \omega \times dm \times \sin\theta = dm\omega x^2 \sin\theta$$

$$L = \int dL = \frac{m\omega L^2}{3} \sin\theta$$

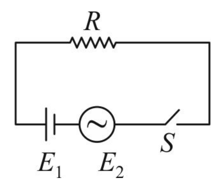
Its horizontal component $= L\cos\theta = \dfrac{m\omega L^2 \sin 2\theta}{6}$

10.(2) Vertical component of $\vec{L}$ remains constant while its horizontal component keeps on changing direction.

$$dL = \frac{m\omega L^2 \sin 2\theta}{6} \times \omega dt\ ; \qquad \frac{dL}{dt} = \frac{m\omega^2 L^2}{6}\sin 2\theta$$

Solution Q. 11 to 12

11.(63) $R = 50\pi$; $\qquad E_1 = 25\sqrt{3}v$



$E_2 = 25\sqrt{6}\sin(\omega t)$; $\qquad \omega = 100\pi s^{-1}$

$E = 25\sqrt{3} + 25\sqrt{6}\sin\omega t$

$$E_{rms} = \sqrt{(25\sqrt{3})^2 + \left[(25\sqrt{6}) \times \frac{1}{\sqrt{2}}\right]^2} \qquad = \sqrt{(25\sqrt{3})^2 + (25\sqrt{3})^2}$$

$$E_{rms} = \sqrt{2(25\sqrt{3})^2} \qquad P = \frac{v^2}{R} = \frac{2(25\sqrt{3})^2}{50} \qquad = \frac{2 \times 25 \times 25 \times 3}{50} \quad = 75\,\text{W}$$

$H = P \times t = 75 \times 14 \times 60 \ = 63000J$

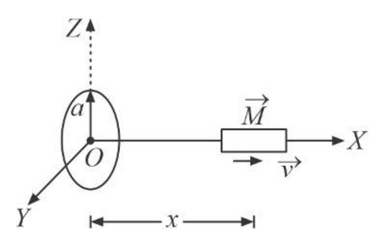
12.(25) $63000 = 3 \times 4200 \times \Delta T$; $\qquad \Delta T = 5°C$

Solution for Q. 13 – 14

13.(D)

14.(B)

The varying magnetic field due to magnetic dipole will produce induced current in the coil. Due to induced current circular conducting loop, it will also behave like a small bar magnet. This bar magnet will produce a force opposing the motion of the magnet. The situation is shown in figure.



The magnetic field at the centre O of the coil due to bar magnet is given as

$$B = \frac{\mu_0}{4\pi} \times \frac{2M}{x^3}$$

Induced EMF induced in circular loop is given as

$$e = -\frac{d\phi}{dt} = -\frac{d}{dt}\left[\frac{\mu_0}{4\pi} \times \frac{2M}{x^3} \times (\pi a^2)\right] \quad \Rightarrow \quad e = -\frac{\mu_0 M a^2}{2}\left(\frac{-3}{x^4} \times \frac{dx}{dt}\right)$$

$$\Rightarrow \qquad e = \frac{3\mu_0 M a^2 v}{2x^4} \text{ where } v = \frac{dx}{dt}$$

Induced current in the circular loop is given as $i = \frac{e}{R} = \frac{3\mu_0 M a^2 v}{2x^4 R}$

If M_{coil} be the magnetic moment of the coil. Then we have $M_{\text{coil}} = i \times A = \frac{3\mu_0 M a^2 v}{2x^4 R} \times \pi a^2$

$$\Rightarrow \quad M_{\text{coil}} = \frac{3\pi\mu_0 M a^4 v}{2x^4 R}$$

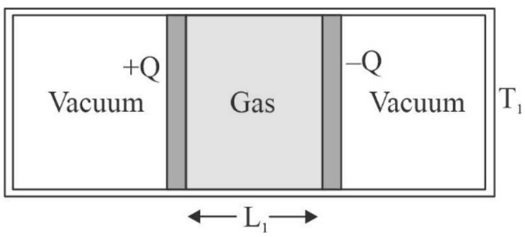
Now, opposing force on bar magnet is given as $F = M\frac{dB}{dx}$

$$\Rightarrow \quad F = \frac{\mu_0}{4\pi}\frac{6M}{x^4}\left[\frac{3\pi\mu_0 M a^4 v}{2x^4 R}\right] \quad \Rightarrow \quad F = \frac{9\mu_0^2 M^2 a^4 v}{4Rx^8}$$

Solution For Q. 15 and 16

15.(A)

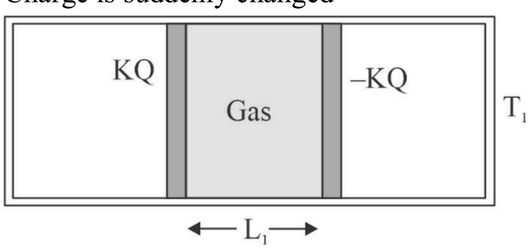
16. (B) Stage-1



$P_1 A L_1 = nRT_1 \quad \frac{Q^2}{2A\varepsilon_0} = P_1 A = \frac{nRT}{L_1}$ (Force Balancing)

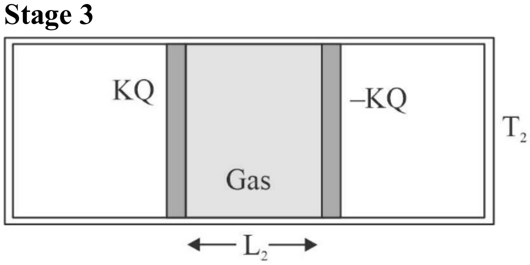
Stage-1

Charge is suddenly changed



System is not in equilibrium

Stage 3



$L_2 C L_1$

Finally system in equilibrium

$$P_2 A L_2 = nRT_2 \quad \frac{K^2 Q^2}{2A\varepsilon_0} = P_2 A = \frac{nRT_2}{L_2}$$

Energy of the system is conserved between stage 2 and 3.

$$\Rightarrow \frac{1}{2}\varepsilon_0 E_2^2 AL_1 + \frac{f}{2}nRT_1 = \frac{1}{2}\varepsilon_0 E_2^2 AL_2 + \frac{f}{2}nRT_2$$

On solving all the above equation

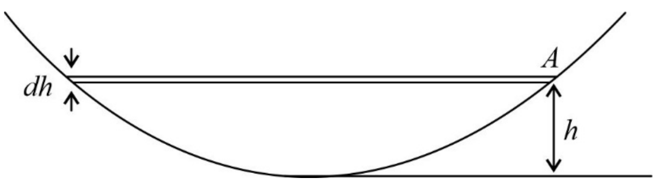
$$\frac{T_2}{T_1} = \frac{2K^2+f}{2+f} \qquad w_{ext} = \left(K^2-1\right)\frac{Q^2L_1}{2A\varepsilon_0} = \left(K^2-1\right)nRT_1$$

17.(64) $E = 2\times\frac{60}{75}\left(\frac{10000}{100}\right) = 160\,V,\ E = E_0\left(\frac{l_1}{l_2}\right)\left(\frac{R_1+R_2}{R_1}\right)$

$$\frac{\Delta E}{E} = \left|\frac{\delta l_1}{l}\right| + \left|\frac{\delta l_2}{l_2}\right| + \left|\frac{\Delta(R_1+R_2)}{R_1+R_2}\right| + \left|\frac{\Delta R_1}{R_1}\right|$$

$$\Delta E = 0.64\,V$$

18.(1)



Let the area of cross section of the bowl at height h be A.

Let the height decrease by dh in further interval dt

Volume that evaporates $= Adh$

As per the question $\frac{Adh}{dt}\alpha A \quad \Rightarrow \frac{Adh}{dt} = -kA$

Where k is a positive constant. We have placed a negative sign because h is decreasing with time and $\frac{dh}{dt}$ is a negative quantity.

$$dh = -k\,dt;\quad \int_{H_0}^{h} dh = -k\int_0^t dt$$

$$h - H_0 = -kt;\ h = H_0 - kt$$

Now $h = \frac{H_0}{2}$ at $t = t_0$ $\therefore$ $\frac{H_0}{2} = H_0 - kt_0$

$$\Rightarrow \frac{H_0}{2} = kt_0 \qquad ....(1)$$

Let height be $\frac{H_0}{4}$ at time 't'

$$\frac{H_0}{4} = H_0 - kt; \qquad \frac{3H_0}{4} = \frac{H_0}{2t_0}t \qquad [u\sin g\,(1)\ k = \frac{H_0}{2t_0}]$$

$$\Rightarrow t = \frac{3}{2}t_0;$$ Required answer is $\frac{t_0}{2}$

19.(37) The energy reaching the sphere is given by

$$E' = \frac{P}{4\pi R^2} \times \pi r^2 = \frac{Pr}{4R^2}$$

Here, $R = 0.8\,m, r = 8\times10^{-3}\,m,\ P = 9.6\times10^{-3}\,W$

$$\Rightarrow \quad E' = \frac{(9.6\times10^{-3})(8\times10^{-3})^2}{4\times(0.8)^2}$$

The number of photoelectrons emitted per second $= E'\times(v/E)$

$$E' = \frac{(9.6\times10^{-3})(8\times10^{-3})^2}{4\times(0.8)^2}\times\frac{10^{-6}}{(5\times1.6\times10^{-19})} \quad \Rightarrow \quad E' = 3\times10^5\,s^{-1}$$

$$KE_{\max} = hv - \phi = 2\,eV$$

After time t, the potential V of the sphere is given by $V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r}\right)$

Where q = charge on the sphere

We also use stopping potential as $V_0 = \frac{E_{\text{photon}} - \phi}{e}$

When potential of sphere becomes 2 volt photoelectric emission stops as maximum kinetic energy of photoelectrons is 2 eV.

$$V_0 = \frac{1}{4\pi\varepsilon_0}\left(\frac{ne}{r}\right) = 2 \quad \Rightarrow \quad \frac{(9\times10^9)\,n(1.6\times10^{-19})}{8\times10^{-3}} = 2$$

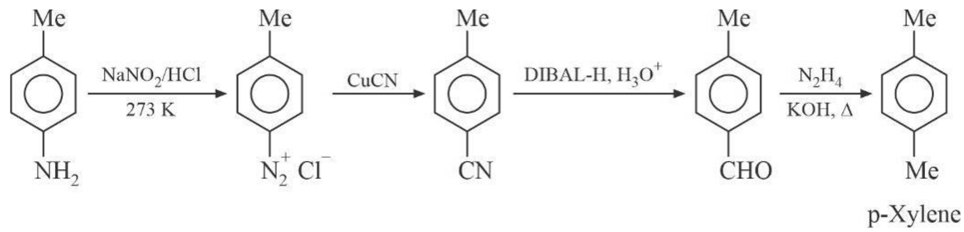
Solving we get ; $\quad n = 1.11\times10^7$

Now, now use $\quad t = \frac{n}{3\times10^5} = \frac{1.11\times10^7}{3\times10^5} \quad \Rightarrow \quad t = 37\text{ sec}$

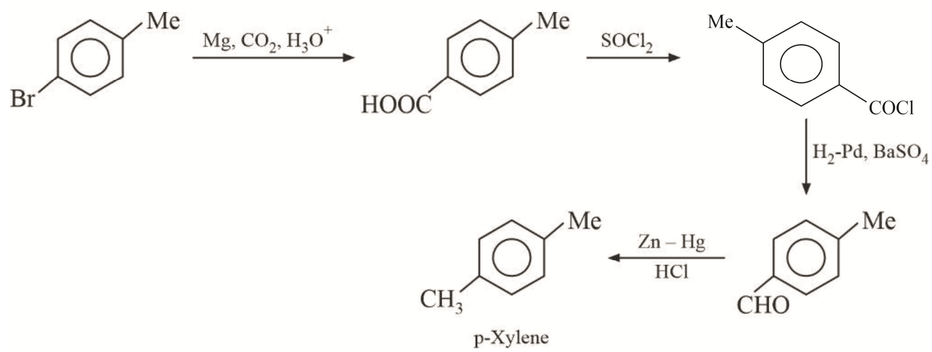
CHEMISTRY

20.(AB)

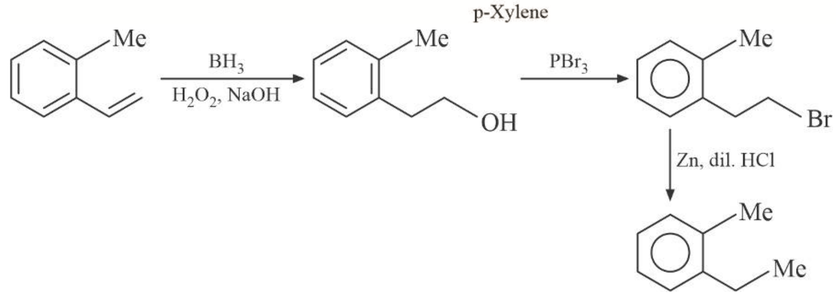
(A)



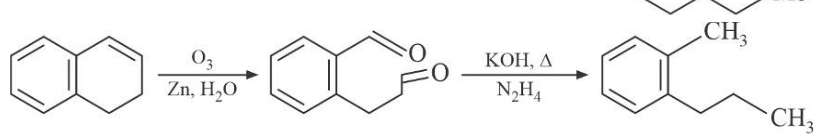
(B)



(C)

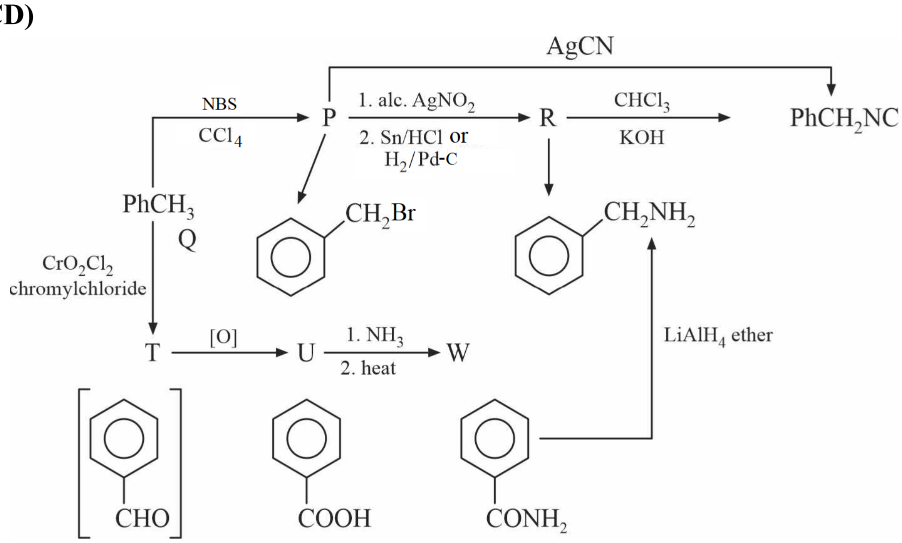


(D)



not a xylene

21.(ACD)



22.(CD)

$$3A(g) \rightarrow 2B(g) + 2C(s)$$

$t = 0$ 6 atm

t $6 - 3P$ $2P$

After 20 mins $P_T = 5$ atm,

$6 - 3P + 2P = 5$; $P = 1$ atm

Pressure of A after 20 mins = 6 – 3P = 3 atm

$\therefore$ $t_{1/2} = 20$ mins ; $t_{75\%} = 2t_{1/2} = 40$ mins

$t_{87.5\%} = 3t_{1/2} = 60$ mins ; $t_{93.75} = 4t_{1/2} = 80$ mins

After 40 mins, $\frac{1}{4}$ A will be left

$P_A = 1.5$ atm ; $P_B = 3$ atm ; $P_T = 4.5$ atm

23.(A) $E^\circ_P = +0.408 + 0.769 > 0$; $E^\circ_Q = +0.161 - 0.769 < 0$

24.(BCD)

(A) $[Cu(NH_3)_4]^{2+}$ $[Cu(Cl)_4]^{2-}$; sp^3

Cu^{2+} $3d^9$

dsp^2

(B) $[Co(CO)_4]^-$; sp^3

$[Co(Cl)_4]^{2-}$; sp^3

(C) $[Ni(Cl)_4]^{2-}$; sp^3

$[Ni(PPh_3)_4]$; sp^3

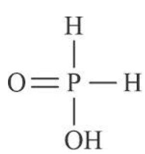
(D) $[Cu(py)_4]^+$; sp^3

$[Cu(CN)_4]^{3-}$; sp^3

25.(ACD)

$$P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$$

sodium hypophosphite



hypophosphorous acid is monobasic and very strong reducing agent

Solution for Q. 26 – 27

26.(6.41)

27.(2.50)

$$\alpha = \frac{\Lambda_m}{\Lambda^\circ_m} = \frac{7}{280} = \frac{1}{40} \; ; \; K_a = \frac{c\alpha^2}{1-\alpha} \quad \Rightarrow \quad K_a = \frac{0.1 \times \left(\frac{1}{40}\right)^2}{\left(1 - \frac{1}{40}\right)} = 6.41 \times 10^{-5}$$

Solution for Q. 28 – 29

28.(7.14)

29.(2.46)

$$Sn + 2HCl \rightarrow SnCl_2 + H_2$$

$$PhNO_2 + 3SnCl_2 + 7HCl \rightarrow PhNH_3^+Cl^- + 3SnCl_4 + 2H_2O$$

So to get 2.58 gm organic salt, we have to form 0.02 mole salts

So, 0.02 mole nitrobenzene is required

0.06 mole Sn is required

So the amount of nitrobenzene = 0.02 × 123 = 2.46 g

The amount of Sn required = 0.06 × 119 = 7.14 g

Solution for Q. 30 – 31

30.(48.88)

31.(9)

Meq. of MnO_2 = Meq. of oxalic acid added – Meq. of oxalic acid left

$$= 1 \times 50 - 10 \times 0.1 \times 32 = 18 \qquad \left\{ \begin{matrix} Mn^{4+} + 2e \rightarrow Mn^{2+} \\ (C^{3+})_2 \rightarrow 2C^{4+} + 2e \end{matrix} \right\}$$

$$\therefore \quad \left[\frac{w}{86.9/2}\right] \times 1000 = 18 \qquad \therefore \quad w_{MnO_2} = \frac{18 \times 86.9}{2000} = 0.7821$$

$$\therefore \quad \% \text{ of } MnO_2 = \frac{0.7821}{1.6} \times 100 = 48.88$$

Also, Meq. of MnO_2 = Meq. of $O_2 = 18$

$$\therefore \quad (w/8) \times 1000 = 18; \quad w_{O_2} = 0.144 \quad \therefore \quad \% \text{ of available } O_2 = \frac{0.144}{1.6} \times 100 = 9\%$$

32.(A) More stable the free radical less is the BDE.

33.(D) Initiation step is endothermic hence option (A) is wrong

Propagation step involving $^{\bullet}CH_3$ formation is endothermic hence option (B) is wrong

Propagation step involving CH_3Br formation is exothermic hence option (C) is wrong

Reaction

$$CH_4 + Br_2 \rightarrow CH_3Br + HBr$$

$$CH_4 \rightarrow CH_3^{\bullet} + H^{\bullet} \qquad \Delta H = 414 \text{ kJ/mol}$$

$$Br_2 \rightarrow Br^{\bullet} + Br^{\bullet} \qquad \Delta H = 193 \text{ kJ/mol}$$

$$Br^{\bullet} + CH_3^{\bullet} \rightarrow CH_3 - Br \qquad \Delta H = -276 \text{ kJ/mol}$$

$$Br^{\bullet} + H^{\bullet} \rightarrow HBr \qquad \Delta H = -368 \text{ kJ/mol}$$

$$CH_4 + Br_2 \rightarrow CH_3 - Br + HBr \qquad \Delta H = -37 \text{ kJ/mol}$$

Overall reaction is exothermic with $\Delta H^\circ = -37 \text{ kJ/mol}$ hence option (D) is correct

$$CH_4 + Br_2 \rightarrow CH_3 - Br + HBr \qquad \Delta H = -37 \text{ kJ/mol}$$

Solution for Q. 34 – 35

34.(B)

35.(A)

$$\underset{A}{CuCO_3} \rightarrow \underset{B}{CuO} + \underset{C}{CO_2}$$

$$CuO + H_2 \rightarrow \underset{D}{Cu} + H_2O$$

$$CuO + H_2SO_4 \rightarrow \underset{E}{CuSO_4} + H_2O$$

$$2CuSO_4 + K_4[Fe(CN)_6] \rightarrow \underset{(F)}{Cu_2[Fe(CN)_6]} + 2K_2SO_4$$

36.(7) Process BC is isothermal

$$W_{BC} = -nRT \ln\frac{V_C}{V_B} \quad = -1 \times R \times 600 \ln\frac{4}{1} \quad = -1200\ R \ln 2$$

In isothermal process $\Delta U = nC_v\Delta T \Rightarrow \Delta U = 0$

According to first law of thermodynamics

$$\Delta U = Q + W = 0 \quad \Rightarrow \quad Q = -W \quad \Rightarrow \quad Q = +1200\ R \ln 2 = 6.9 \times 10^3\ J$$

37.(198)

$$\lambda = \frac{h}{p},\ P = \frac{h}{\lambda},\ \frac{6.6 \times 10^{-34}}{100 \times 10^{-9}} = \frac{2 \times 10^{-3}}{6 \times 10^{23}} \times V$$

1.98 m/sec

38.(3) Oxidation state of ‘Pt’ in $XePtF_6$ is +5

Oxidation state of ‘Xe’ in Ba_2XeO_6 is +8

MATHEMATICS

39.(ACD)

Let $f(x) = (1+x)(1+x^2)\ldots\ldots(1+x^{2070})$

Then A must be the sum of coefficients of x^{9k} where k is non-negative integer

Let $\alpha = e^{i2\pi/9}$

Note: $\alpha^3 = w$ and $\alpha^6 = w^2$, where w and w^2 are complex cube roots of unity.

Putting $x = \alpha, \alpha^2, \alpha^4, \alpha^5, \alpha^7, \alpha^8$ in :

$(1+x)(1+x^2)\ldots..(1+x^{2070})$ gives:

$\left[(1+\alpha)(1+\alpha^2)\ldots.(1+1)\right]^{230} = 2^{230}$ because $(1+\alpha)\ldots..(1+\alpha^8) = 1$

Also, putting : $x = \alpha^3, \alpha^6$ in :

$(1+x)(1+x^2)\ldots\ldots(1+x^{2070})$ gives: $\left((1+w)(1+w^2)2\right)^{690} = 2^{690}$

Putting $x = 1$ in :

$(1+x)(1+x^2)\ldots\ldots(1+x^{2070})$ gives 2^{2070}

Adding all these, we get:

$$9A = 2^{2070} + 2.2^{690} + 6.2^{230} \quad \Rightarrow \quad A = \frac{1}{9}2^{231}\left(2^{1839} + 2^{460} + 3\right)$$

40.(ABCD)

$$\tan\frac{B}{2} = \frac{2}{3}, \tan\frac{C}{2} = \frac{1}{2} \quad \Rightarrow \quad \tan\frac{A}{2} = \frac{4}{7}$$

$$\tan\frac{B}{2}\cdot\tan\frac{C}{2} = \frac{s-a}{s} \quad \Rightarrow \quad 2s = 3a = 42$$

$\therefore$ Perimeter = 42 $\quad\therefore\quad \Delta = r.s. = 84\ cm^2$

$\therefore$ $\tan\frac{A}{2}, \tan\frac{B}{2}, \tan\frac{C}{2}$ all are less than 1. All angles are acute.

41.(CD)

$$g(x) = \lim_{n\to\infty} n\left[x^{\frac{2018}{n}} - x^{\frac{2019}{n}}\right]$$

Let $n = \frac{1}{t}$ as $n \to \infty,\ t \to 0$; $\quad g(x) = \lim_{t\to 0}\frac{x^{2018t} - x^{2019t}}{t}\left(\frac{0}{0}\right)$

Apply L' hospital Rule $g(x) = -\ln(x)$

42.(AC) $y = \lambda_1\cos 2x + \lambda_2$

$\Rightarrow$ Differential equation is $(\sin 2x)\dfrac{d^2y}{dx^2} = 2\cdot\dfrac{dy}{dx}\cdot\cos 2x \quad \Rightarrow \quad \lambda = 0, f(x) = \sin 2x$

43.(AD) Given

$(\vec{a}+\vec{b})\times(4\hat{i}+\lambda\hat{j}+2\hat{k})=0$

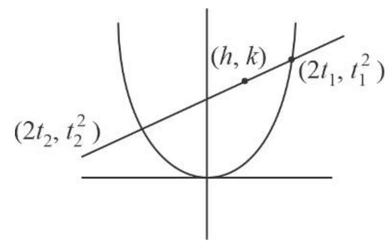
$|\vec{a}+\vec{b}||-3\hat{i}+2\hat{j}+\lambda\hat{k}|\cos\theta=10$

θ is the angle between $(4\hat{i}+\lambda\hat{j}+2\hat{k})$ and $(-3\hat{i}+2\hat{j}+\lambda\hat{k})$

$\therefore \quad 7\lambda^2-48\lambda+52=0$

44.(ABD)

$\frac{t_1^2-t_2^2}{2(t_1-t_2)}=\frac{1}{2} \quad \Rightarrow \quad t_1+t_2=1$



Also, $h=\frac{4t_1+2t_2}{3}, k=\frac{2t_1^2+t_2^2}{3}$

$\Rightarrow \quad 3h=4t_1+2(1-t_1), 3k=2t_1^2+(1-t_1)^2$

$t_1=\frac{3h-2}{2}, 3k=3t_1^2-2t_1+1;\ 3k=3\left(\frac{3h-2}{2}\right)^2-2\left(\frac{3h-2}{2}\right)+1$

$\Rightarrow \quad \left(h-\frac{8}{9}\right)^2=\frac{4}{9}\left(k-\frac{2}{9}\right) \Rightarrow \quad a=8,\ b=2,\ c=4$

Solution for Q. 45 – 46

45.(13)

46.(36)

$C_1:\frac{(x-3)^2}{9}+\frac{(y-2)^2}{4}=1 \quad \therefore \quad A_1=(6, 2), A_2=(0, 2) \quad \therefore \quad |PA_1-PA_2|=3\sqrt{2}$

Clearly locus of P is hyperbola for which $A_1A_2=2ae=6$ and $2a=3\sqrt{2}\Rightarrow e=\sqrt{2}$

$\therefore$ Lotus of P is rectangular hyperbola

Equation of conic $C_2:(x-3)^2-(y-2)^2=\frac{9}{2}$

Now, $D_1=2ae=6$; $\quad D_2=b^2=\frac{9}{2}$

$D_3:L=\sqrt{9-\frac{9}{2}}=\frac{3}{\sqrt{2}} \quad \therefore \quad \left(\frac{D_1D_2}{D_3^2}\right)^2=36$

47.(0) $\lim_{x\to 1}\frac{f(x)}{(x-1)^2}=1$

$\Rightarrow \quad f(1)=0, f'(1)=0, f''(1)=2$

$\therefore \quad f(x)=A(x-1)^4+B(x-1)^3+(x-1)^2$

$f'(x)=4A(x-1)^3+3B(x-1)^2+2(x-1)$

$f'(0)=-6\Rightarrow -4A+3B-2=-6 \quad \ldots$ (i)

$f'(2) = 6 \Rightarrow 4A + 3B + 2 = 6$ … (i)

(i), (ii) $\Rightarrow$ $A = 1, B = 0$

$\therefore$ $f(x) = (x-1)^4 + (x-1)^2 \geq 0$

$f(1) = 0$ is the minimum

48.(27) $f(x) = (x-1)^4 + (x-1)^2, f'(0) = -6, f(0) = 2$

$f'(x) = 4(x-1)^3 + 2(x-1)$

Subtangent at $x = 0$ is $\left|\frac{f(0)}{f'(0)}\right| = \frac{2}{6} = \frac{1}{3} = l$

$81l = \frac{81}{3} = 27$

Solution for Q. 49 – 50

49.(4)

50.(18)

Solution for Q. 49 – 50

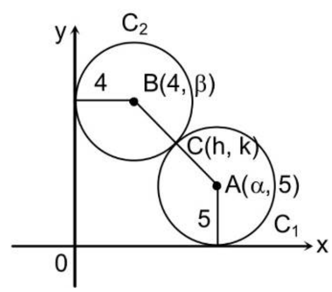
$I_1 = 2\int_1^{10^2} \{t\}dt$; $\left[\text{Put } \sqrt{x} = t, \frac{dx}{2\sqrt{x}} = dt\right]$; $I_2 = \int_1^{10} (x\{x^2\})\, dx$ $[x^2 = N, 2x\, dx = dN]$

$I_2 = \int_1^{10^2} \frac{\{N\}dN}{2}$; $\frac{I_1}{I_2} = 4,$ $I_1 = 2 \times 99\int_0^1 \{t\}\, dt = 198\int_0^1 t\, dt = 99$

Solution for Q. 51 – 52

51.(D)

52.(B)



$C(h, k)$ divides AB into the ratio 5 : 4 $\therefore$ $(h, k) = \left(\frac{4\alpha + 20}{9}, \frac{5\beta + 20}{9}\right)$

$AB = 9 \Rightarrow$ $(\alpha - 4)^2 + (\beta - 5)^2 = 9^2$

$\Rightarrow$ $\frac{(h-4)^2}{4^2} + \frac{(k-5)^2}{5^2} = 1$

Locus of point of contact is $\frac{(x-4)^2}{16} + \frac{(y-5)^2}{25} = 1$

We shift the origin to (4, 5), so that curve become $\frac{x^2}{16} + \frac{y^2}{25} = 1$ and point (9, 5) becomes (5, 0)

Chord of contact is $x = \frac{16}{5}$

Solving with conic, we get Q and R as $\left(\frac{16}{5}, \pm 3\right)$ $\therefore$ Area $\Delta PQR = \frac{27}{5}$

53.(A) $f'(x) = x\dfrac{\left(1+\dfrac{x}{\sqrt{1+x^2}}\right)}{x+\sqrt{1+x^2}} + \ln(x+\sqrt{x^2+1}) - \dfrac{x}{\sqrt{x^2+1}}$

$= \dfrac{x}{\sqrt{x^2+1}} + \ln(x+\sqrt{x^2+1}) - \dfrac{x}{\sqrt{x^2+1}}$

$= \ln(x+\sqrt{x^2+1})$ $\quad > 0 \ \forall\ x > 0$

$\quad < 0 \ \forall\ x < 0$

$\therefore$ $\quad h'(x) = (1-2f(x)+3f^2(x))f'(x) \quad = f'(x)\,(1-2f(x)+3f^2(x))$

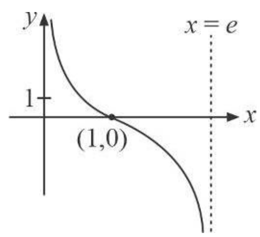
$D < 0$

$\Rightarrow$ $h(x)$ is increasing when f is increasing and decreasing when f is decreasing

$\therefore$ $h(x)$ increasing in $(0, \infty)$ and $h(x)$ decreasing in $(-\infty, 0)$

54.(D) $g(x) = \ln(1-\ln x)$

Domain $(0, e)$



$g'(x) = -\dfrac{1}{(1-\ln x)}\cdot\dfrac{1}{x} < 0 \Rightarrow$ decreasing $\forall\ x$ in its domain $\Rightarrow$ (A) and (B) are incorrect

$g'(1) = -1 \quad \Rightarrow \quad$ (C) is also incorrect

Also $\quad g(1) = 0,\ \lim\limits_{x\to e^-} g(x) \to -\infty,\ \lim\limits_{x\to 0^+} g(x) \to \infty$

$$g''(x) = \frac{-\ln x}{x^2(1-\ln x)^2}$$

$g''(1) = 0$ which is a point of inflection as shown in graph y-axis and $x = e$ are two asymptotes

55.(5) Let R be the event that a red face appears in each of the first n throws,

E_1 : Die A is used when head has already fallen

E_2 : Die B is used when tail has already fallen

$\therefore \quad P\left(\dfrac{R}{E_1}\right) = \left(\dfrac{2}{3}\right)^n \ \&\ P\left(\dfrac{R}{E_2}\right) = \left(\dfrac{1}{3}\right)^n; \qquad \dfrac{P(E_1)\cdot P\left(\dfrac{R}{E_1}\right)}{P(E_1)\cdot P\left(\dfrac{R}{E_1}\right) + P(E_2)\cdot P\left(\dfrac{R}{E_2}\right)} = \dfrac{32}{33}$

$$\frac{\frac{1}{2}\cdot\left(\frac{2}{3}\right)^n}{\frac{1}{2}\left(\frac{2}{3}\right)^n + \frac{1}{2}\left(\frac{1}{3}\right)^n} = \frac{32}{33}; \qquad n = 5$$

56.(3) Let $h = 4 + 4\cos\phi,\ k = 3 + 3\sin\phi$

Reflection of (h, k) about line $x - y - 2 = 0$ is

$h' = 5 + 3\sin\phi$ and $k' = 2 + 4\cos\phi$

Equation of curve after reflection is

$$\left(\frac{x-5}{3}\right)^2 + \left(\frac{y-2}{4}\right)^2 = 1$$

$$16x^2 + 9y^2 - 160x - 36y + 292 = 0$$

$$k_1 + k_2 = 132 = 2^2 \cdot 3 \cdot 11$$

57.(2) $f(x) = \dfrac{x}{1+x^2}$

$$\Rightarrow \int_{-\pi/4}^{\pi/4} \frac{f(x) + x^9 - x^3 + x + 1}{\cos^2 x}\, dx = \int_{-\pi/4}^{\pi/4} \sec^2 x\, dx = 2$$